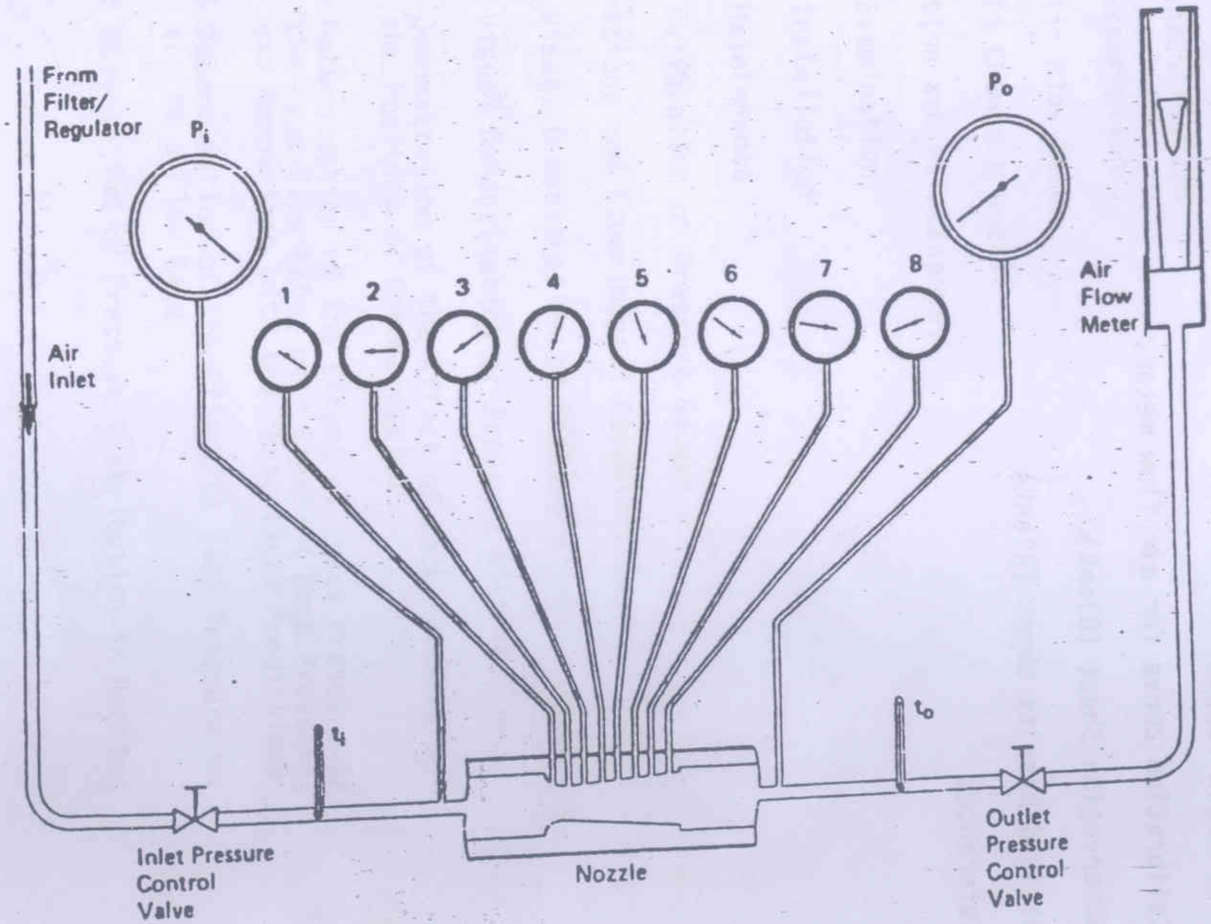
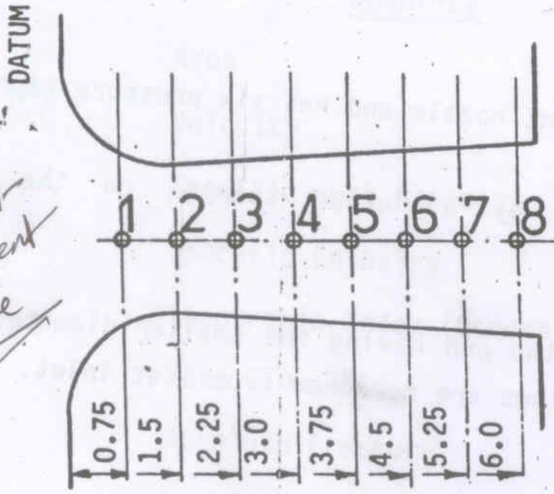
Nozzle Pressure Distribution Unit



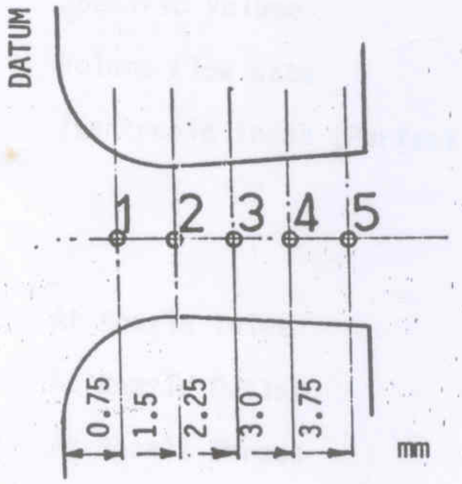
NOZZLE PROFILES

Convergent-Divergent Nozzle



Nozzle A

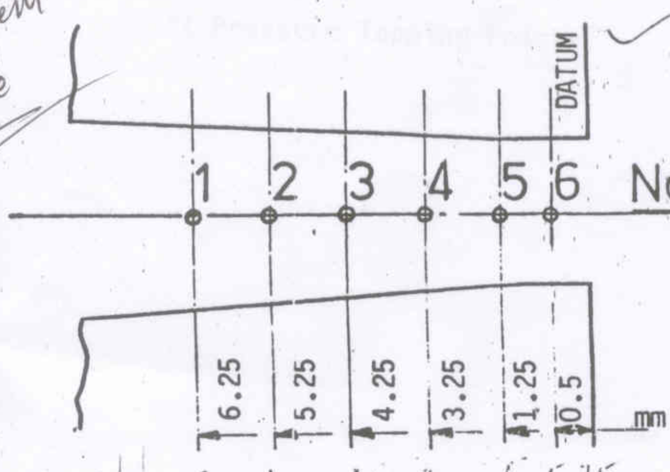
Tapping Point	Nominal Diameter mm	Area / Throat Area
1	2.4	1.44
2	2.0	1.0
3	2.13	1.13
4	2.26	1.28
5	2.39	1.42
6	2.52	1.59
7	2.66	1.77
8	2.79	1.94



Nozzle B

1	2.4	1.44
2	2.0	1.0
3	2.13	1.13
4	2.26	1.28
5	2.39	1.42

Convergent Nozzle



Nozzle C

1	2.86	2.05
2	2.65	1.75
3	2.43	1.48
4	2.21	1.2
5	2.03	1.03
6	2.0	1.0

OBSERVATION SHEET

Date:

Nozzle: C

Atmospheric Pressure: 102 KN m^{-2}

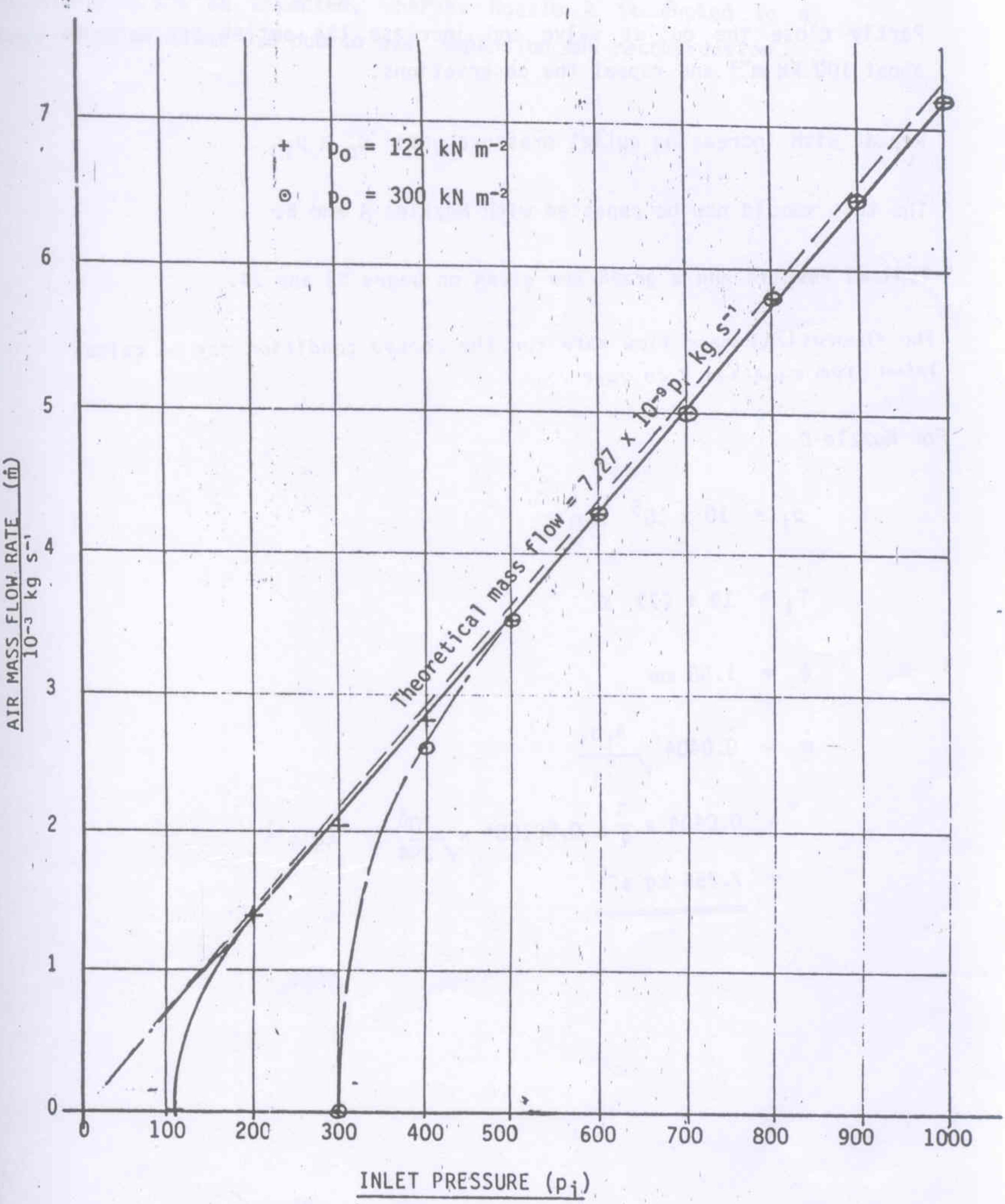
Throat Diameter: 1·98 mm

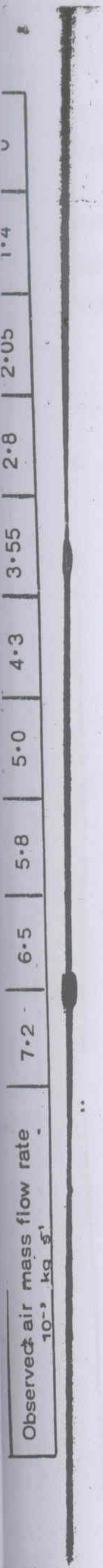
Inlet Temperature	(t_i)	°C	19					Constant				
	(T_i)	K	292					Constant				
Outlet Temperature		°C	18					Constant				
pressure kN m^{-2}	Inlet (p_i)	gauge	900	800	700	600	500	400	300	200	100	20
		abs	1002	902	802	702	602	502	402	302	202	122
	Outlet (p_o)	gauge	20					Constant				
		abs	122					Constant				
	Tapping No. 1 (p_1)	gauge										
		abs										
	" 2 (p_2)	gauge										
		abs										
	" 3 (p_3)	gauge										
		abs										
	" 4 (p_4)	gauge										
		abs										
	" 5 (p_5)	gauge										
		abs										
	" 6 (p_6)	gauge										
		abs										
	" 7 (p_7)	gauge										
		abs										
	" 8 (p_8)	gauge										
		abs										
[illegible] air mass flow rate			7·2	6·5	5·8	5·0	4·3	3·55	2·8	2·05	1·4	0

AIR MASS FLOW RATE ($\dot{m}$)

RELATIONSHIP BETWEEN INLET PRESSURE AND AIR MASS FLOW RATE

(Inlet Temperature 292 K)





OBSERVATION SHEET

Date:

Nozzle: C

Atmospheric Pressure: 99·3 KN m^{-2}

Throat Diameter: 1·98 mm

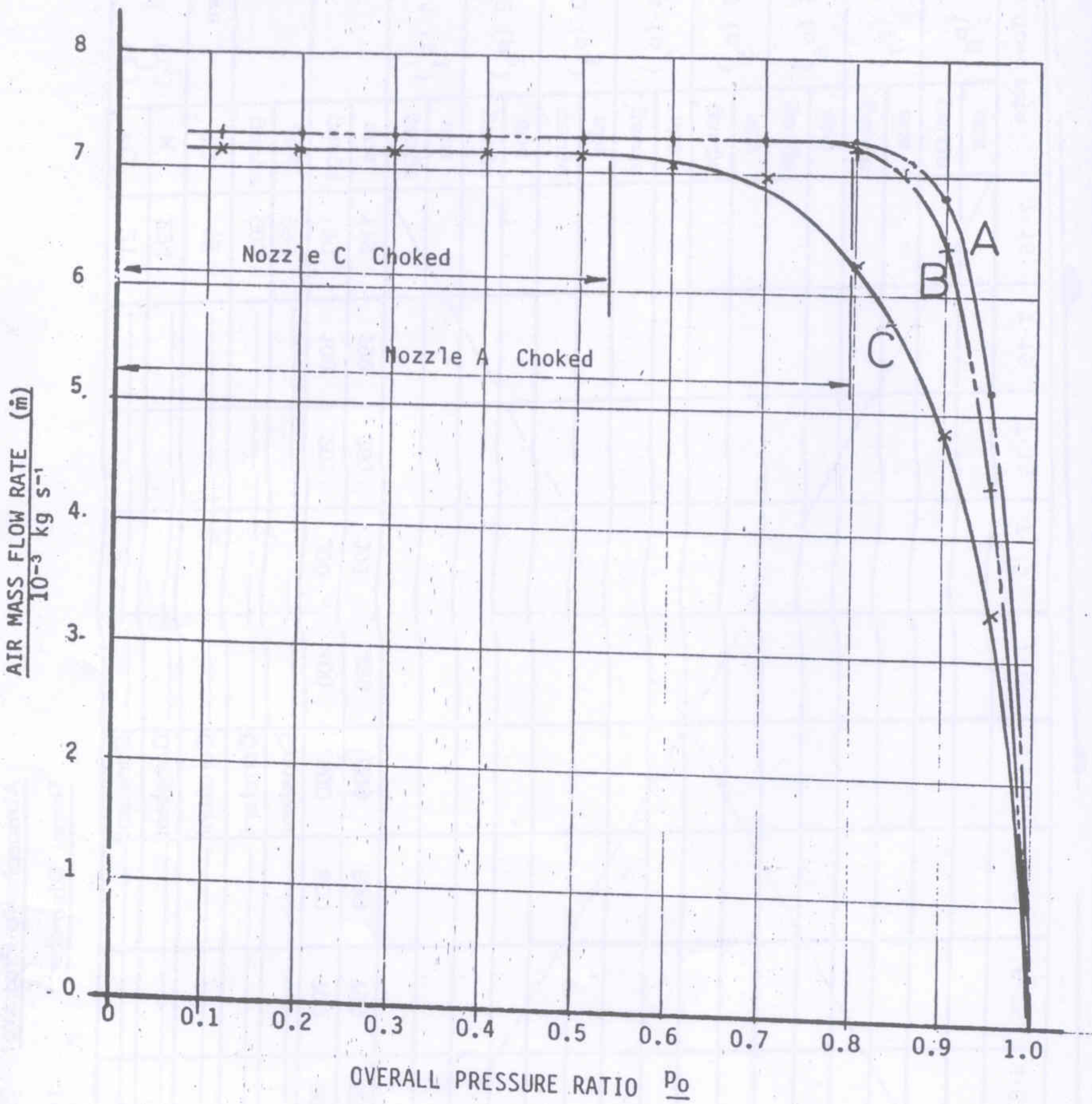
Inlet Temperature	(t_i)	°C	21	—	—	—	—	Constant	—	—	—	—
	(T_i)	K	294	—	—	—	—	Constant	—	—	—	—
Outlet Temperature		°C	19	—	—	—	—	Constant	—	—	—	—
pressure kN m^{-2}	Inlet (p_i)	gauge	900	—	—	—	—	Constant	—	—	—	—
		abs	999	—	—	—	—	Constant	—	—	—	—
	Outlet (p_o)	gauge	20	100	200	300	400	500	600	700	800	850
		abs	119	199	299	399	499	599	699	799	899	949
	Tapping No. 1 (p_1)	gauge										
		abs										
	" 2 (p_2)	gauge										
		abs										
	" 3 (p_3)	gauge										
		abs										
	" 4 (p_4)	gauge										
		abs										
	" 5 (p_5)	gauge										
		abs										
	" 6 (p_6)	gauge										
		abs										
	" 7 (p_7)	gauge										
		abs										
	" 8 (p_8)	gauge										
		abs										
Observed air mass flow rate 10^{-3} kg s^{-1}			7·15	7·15	7·15	7·15	7·15	7·10	7·0	6·25	4·84	3·4

AIR MASS FLOW RATE ($\dot{m}$) 10^{-3} kg s^{-1}

RELATIONSHIP BETWEEN AIR MASS FLOW RATE AND OVERALL PRESSURE RATIO

Inlet Pressure: 1000 kN m⁻²

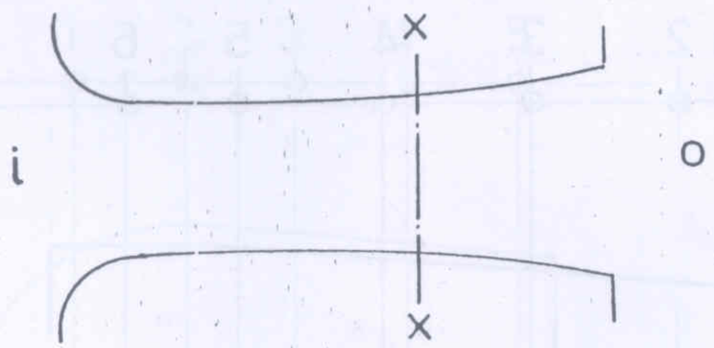
Inlet Temperature: 294 K



APPENDIX

THEORY

Flow through a nozzle



For reversible and adiabatic one dimensional expansion through a passage, the following relationships apply at any section XX.

$$C_x = \sqrt{2(h_i - h_x)} \qquad A$$

$$A_x = \frac{\dot{m}v_x}{C_x} \qquad B$$

If it is assumed that the relationship between p and v in such an expansion is $pv^k =$ constant,

$$h_i - h_x = \int_i^x vdp$$

$$= \frac{k}{k-1} p_i v_i \left[1 - \left(\frac{p_x}{p_i}\right)^{\frac{k-1}{k}}\right] \qquad C$$

and substituting this in A,

$$C_x = \sqrt{\frac{2k}{k-1} p_i v_i \left[1 - \left(\frac{p_x}{p_i}\right)^{\frac{k-1}{k}}\right]} \qquad D$$

Also, $$v_x = \left(\frac{p_i}{p_x}\right)^{\frac{1}{k}} v_i$$

and, substituting for C_x and v_x in B,

$$A_x = \frac{\dot{m}\left(\frac{p_i}{p_x}\right)^{\frac{1}{k}} v_i}{\sqrt{\frac{2k}{k-1} p_i v_i \left[1 - \left(\frac{p_x}{p_i}\right)^{\frac{k-1}{k}}\right]}}$$

Rearranging,

$$\frac{\dot{m}}{A_x} = \sqrt{\frac{2k}{k-1} \frac{p_i}{v_i} \left[\left(\frac{p_x}{p_i}\right)^{\frac{2}{k}} - \left(\frac{p_x}{p_i}\right)^{\frac{k+1}{k}}\right]} \qquad E$$

This expression has a maximum value, when

$$\frac{p_x}{p_i} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$ (This is called the Critical Pressure Ratio)

Since $\dot{m}$ is a constant, $\frac{\dot{m}}{A_x}$ is a maximum when A_x has its smallest value, i.e. at the "throat".

Thus, $$\frac{p_t}{p_i} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \qquad F$$

If this value is substituted in D,

$$C_t = \sqrt{\frac{2k}{k+1} p_i v_i} \qquad G$$

or $$C_t = \sqrt{k\, p_t v_t}$$ which is the local speed of sound.

Substituting the throat conditions into equation B, i.e.

$$C_t = \sqrt{\frac{2k}{k+1} p_i v_i}$$

and

$$v_t = \left(\frac{2}{k+1}\right)^{\frac{-1}{k-1}} v_i$$

we obtain

$$\frac{\dot{m}}{A_t} = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \sqrt{\frac{2k}{k+1} \frac{p_i}{v_i}} \qquad \text{H}$$

If Equation H is divided by E we obtain the relationship between the throat area and the area at any section x.

$$\frac{A_x}{A_t} = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \sqrt{\frac{k-1}{(k+1)\left[\left(\frac{p_x}{p_i}\right)^{\frac{2}{k}} - \left(\frac{p_x}{p_i}\right)^{\frac{k+1}{k}}\right]}} \qquad \text{J}$$

and for nozzle exit,

$$\frac{A_o}{A_t} = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \sqrt{\frac{k-1}{(k+1)\left[\left(\frac{p_o}{p_i}\right)^{\frac{2}{k}} - \left(\frac{p_o}{p_i}\right)^{\frac{k+1}{k}}\right]}} \qquad \text{K}$$

From the foregoing it will be seen that when a compressible fluid expands reversibly and adiabatically in a passage through a pressure ratio

$$\frac{p_o}{p_i} < \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

(i) The passage must have reducing cross-sectional area, i.e. converge, until

$$\frac{p_t}{p_i} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

and must then have increasing cross-sectional area, i.e. diverge.

(ii) At the minimum area in the passage, i.e. the throat, the velocity of the fluid is the local speed of sound.

(iii) In the converging portion, velocity will be subsonic and in the diverging portion will be supersonic.

(iv) The mass flow rate through the passage is determined by the cross-sectional area of the throat and the properties of the fluid at inlet. It is not affected by the value of p_2 as long as the critical pressure ratio is maintained at the throat.

If the fluid flowing is a perfect gas we may use the following relationships:

$$p_v = RT$$

$$R = C_p - C_v$$

$$\gamma = \frac{C_p}{C_v}$$

$$h_1 - h_2 = C_p(T_1 - T_2)$$

and during a reversible and adiabatic process $p_v \gamma =$ Const, etc.

For air at the conditions met in this unit, we may assume that it behaves as a perfect gas with

$$R = 287.1 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$\gamma = 1.4$$

$$C_p = 1005 \text{ J kg}^{-1} \text{ K}^{-1}$$

Using these relationships, Equation C becomes

$$C_x = \sqrt{\frac{2\gamma}{\gamma - 1} RT_i \left[1 - \left(\frac{p_x}{p_i}\right)^{\frac{\gamma - 1}{\gamma}}\right]}$$

$$= \sqrt{\frac{2 \times 1.4 \times 287.1}{0.4}} \sqrt{T_i \left[1 - \left(\frac{p_x}{p_i}\right)^{\frac{0.4}{1.4}}\right]}$$

$$C_x = 44.83 \sqrt{T_i \left[1 - \left(\frac{p_x}{p_i}\right)^{0.286}\right]} \text{ m s}^{-1} \qquad \text{L}$$

Equation H becomes

$$\dot{m} = A_t \left(\frac{2}{\gamma + 1}\right)^{\frac{1}{\gamma - 1}} \sqrt{\frac{2\gamma}{\gamma + 1} \frac{p_i^2}{RT_i}}$$

$$= A_t \left(\frac{2}{2.4}\right)^{\frac{1}{0.4}} \sqrt{\frac{2 \times 1.4}{2.4 \times 287.1}} \frac{p_i}{\sqrt{T_i}}$$

$$\dot{m} = 0.0404 \frac{A_t p_i}{\sqrt{T_i}} \text{ kg s}^{-1} \qquad \text{M}$$

Equation G becomes

$$C_t = \sqrt{\frac{2\gamma}{\gamma + 1} RT_i}$$

$$= \sqrt{\frac{2 \times 1.4 \times 287.1}{2.4} \times T_i}$$

$$C_t = 18.3 \sqrt{T_i} \quad m\ s^{-1} \qquad N$$

It should be appreciated that the above relationships apply to reversible and adiabatic, i.e. isentropic, flow.

Although flow through practical nozzles may usually be assumed to be adiabatic, there will be various losses due to friction and shock - particularly in the divergent portion - which will render the expansion irreversible.